FUNDAMENTAL MODE PROPAGATION ON DIELECTRIC FIBERS OF SOME NONCIRCULAR CROSS SECTIONS

Edward F. Kuester and Ronald C. Pate
Electromagnetics Laboratory
Department of Electrical Engineering
University of Colorado
Boulder, Colorado 80309

ABSTRACT

The behavior of the propagation constants of the fundamental modes on dielectric fibers of arbitrary cross-section is studied by a variational formulation of the integral equation for the modal electric field. An approximate eigenvalue equation is obtained which involves functions defined as series expansions easily and accurately evaluated by computer, into which the cross-sectional shapes enter only through a set of constants calculated ahead of time. These constants are calculable analytically for elliptical, polygonal, and several other shapes, and used to obtain dispersion curves which compare favorably in the single-mode regime with results for rectangular, elliptical and triangular cores computed by other methods.

Introduction

Optical dielectric waveguides (fibers) of various core shapes have been fabricated or proposed for use for a number of applications in optical communications systems, generally because of some desirable propagation characteristic or for reasons of ease of fabrication.[1,2] However, analytical solutions can be obtained only for cross-sections with circular symmetry, while other shapes must be analyzed numerically or by analytical approximation.

Thus the exact solution of the elliptic rod[3-5] involves evaluation of an infinite determinant containing Mathieu functions; the rectangular channel (probably the most widely studied of the noncircular shapes) has prompted elaborate numerical schemes[6-11] and approximate techniques[12-14] in an effort to determine its dispersion curves. A fiber with triangular core has also been investigated numerically.[15] There are also quite a few methods in the literature applicable to guides of arbitrary cross section using such means as point-matching,[16] moment methods,[10] or finite element methods.[17-18]

A very promising approach can be based on an integral equation for the electric field of a guided mode due to Katsenelenbaum.[20] As suggested by Shaw et al.,[21,22] this integral equation can be used to generate a variational expression in which the unknown propagation constant appears implicitly. Unfortunately the numerical implementation used by these authors required repeated evaluations of double surface integrals (i.e., quadruple integrals) for different values of propagation constant until the correct value was found; a number of anomalous results were obtained and it appeared uncertain as to how the errors could be reduced. In this paper a new implementation of this approach is suggested, which leads to a simple method for obtaining dispersion properties of the fundamental modes in the single-mode range and avoids the difficulties encountered by previous workers.

Derivation of Approximate Mode Equation

Consider the dielectric rod of cross section S (by which we also denote the cross-sectional area) located along the Z-axis in a homogeneous region (cladding) of infinite extent and permittivity $\varepsilon_c = \varepsilon_2$. As shown in Fig. 1, the rod (core) has a permittivity $\varepsilon_1 = \varepsilon_r \varepsilon_c$, where ε_r can be a function of the transverse coordinates (x,y) within the core. We choose the diameter d of the rod (the largest distance between two points in S) as a convenient scale dimension of the problem. We seek mode solutions for this rod, decaying away from the rod in the transverse direction, and propagating according to the factor $\exp(i\omega t - ik_c \nu z)$, where $k_c = \omega\sqrt{\mu_0 \varepsilon_c}$ and ν is a normalized propagation constant which we wish to determine.

The E-field integral equation[20] can be written in operator form as

$$\bar{E}(\bar{r}_t) = M_\nu[\Delta(\bar{r}_t')\bar{E}(\bar{r}_t')] \tag{1}$$

where $\bar{r}_t = (x,y)$ and $\bar{r}_t' = (x',y')$ denote transverse position vectors,

$$\Delta(\bar{r}_t) = \varepsilon_r(\bar{r}_t) - 1 \tag{2}$$

and

$$M_\nu[\bar{F}(\bar{r}_t')] = -\frac{1}{2\pi}[\text{grad div} + k_c^2]\int\int_{-\infty}^{\infty} \bar{F}(\bar{r}_t')K_0(\gamma k_c|\bar{r}_t - \bar{r}_t'|)d\bar{r}_t' \tag{3}$$

for any vector function $\bar{F}(\bar{r}_t)$. The "grad" and "div" operators are understood to have their z-derivatives replaced by the algebraic factor $-ik_c\nu$, while K_0 is the modified Bessel function, and

$$\gamma = (\nu^2 - 1)^{\frac{1}{2}} \tag{4}$$

By employing the usual operator and inner product arguments[21] we arrive at the following expression for ν, stationary with respect to the electric field $\bar{E}$:

$$\langle \Delta\bar{E}^T, \bar{E}\rangle - \langle \Delta\bar{E}^T, M_\nu[\Delta\bar{E}]\rangle = 0 \tag{5}$$

where the inner product is defined as

$$\langle \bar{F}, \bar{G}\rangle \equiv \int\int_{-\infty}^{\infty} \bar{F}(\bar{r}_t)\cdot\bar{G}(\bar{r}_t)d\bar{r}_t \tag{6}$$

and

$$\bar{E}_t^T = \bar{E}_t \;; \qquad E_z^T = -E_z \tag{7}$$

for any function $\bar{E}$. Note that all integrals contained in (5) are actually carried out only over the cross section S, since $\Delta(\bar{r}_t) \equiv 0$ outside this region. Equation (5) is analogous to the Schwinger variational principle in quantum mechanics.

We shall restrict ourselves here to weakly-guiding, step-index fibers for which

$$\Delta = \varepsilon_r - 1 = \text{const} \ll 1 \tag{8}$$

Under these conditions, the longitudinal field components are known to be small compared to the transverse fields.[23] Moreover, inspection of existing results for various core shapes reveals that the transverse electric fields inside the core are nearly constant for the fundamental modes close enough to cutoff. Based upon these observations, we choose as our trial field

$$E_x = \alpha_1; \quad E_y = \alpha_2; \quad E_z = 0 \tag{9}$$

with α_1 and α_2 as adjustable constants.

Substituting (9) into the variational expression and applying the stationary conditions ($\partial/\partial\alpha_1 = 0$, $\partial/\partial\alpha_2 = 0$) we obtain the approximate eigenvalue equations

$$\frac{1+\Delta/2}{k_c^2 S\Delta} = (1+\frac{\gamma^2}{2})Q \pm \sqrt{\left(\frac{Q_{xx}-Q_{yy}}{2}\right)^2 + Q_{xy}^2} \tag{10}$$

where

$$Q(\nu) = \frac{Q_{xx}(\nu) + Q_{yy}(\nu) - 1/k^2 S}{\gamma^2} \tag{11}$$

$$Q_{ss'}(\nu) = \frac{1}{2\pi k_c^2 S^2} \int_S \frac{\partial^2}{\partial s \partial s'} \int_S K_0(\gamma k_c |\bar{r}_t - \bar{r}_t'|) d\bar{r}_t' d\bar{r}_t \tag{12}$$

and s,s' stand for x or y. It can be seen that two separate eigenvalue equations are found unless $Q_{xy} = 0$ and $Q_{xx} = Q_{yy}$, corresponding to two fundamental modes with distinct polarizations. In fibers of high symmetry (e.g., the circular fiber), these two modes will be degenerate. If only $Q_{xy} = 0$, the fundamental modes are polarized in the x- and y-directions if $Q_{xx} \neq Q_{yy}$, and in any two orthogonal directions if $Q_{xx} = Q_{yy}$.

The double surface integrals in (11) and (12) can be reduced to double line integrals by applications of the divergence theorem. In terms of these, the eigenvalue equations (10) become

$$\frac{1+\frac{\Delta}{2}}{\Delta} = \frac{1+\frac{\gamma^2}{2}}{\gamma^2}[1 - T_{xx} - T_{yy}] \pm \sqrt{\left(\frac{T_{xx}-T_{yy}}{2}\right)^2 + T_{xy}^2} \tag{13}$$

where

$$T_{ss'}(\nu) = \frac{1}{2\pi S} \oint_C \oint_C (\bar{a}_n \cdot \bar{a}_s)(\bar{a}_n' \cdot \bar{a}_{s'}) K_0(\gamma k_c |\bar{r}_t - \bar{r}_t'|) d\ell' d\ell$$

$$= \frac{1}{2\pi} \sum_{n=0}^{\infty} \frac{1}{(n!)^2} \left(\frac{\gamma k_c d}{2}\right)^{2n} \left\{ [\Psi(n) - \ell n(\frac{\gamma k_c d}{2})] T_{2n}^{ss'} - N_{2n}^{ss'} \right\} \tag{14}$$

Here C is the boundary of S, $\bar{a}_n$ and $\bar{a}_n'$ are unit normal vectors to C and $\bar{x}_t$ and $\bar{x}_t'$ respectively. The second form of (14) is obtained by replacing K_0 by its small-argument series expansion and integrating termwise. We have used the notations

$$\Psi(n) = -C + \sum_{k=1}^{n-1} k^{-1} \quad ; \quad \Psi(0) = -C = -0.5772 \ \ldots \tag{15}$$

$$T_{2n}^{ss'} = \frac{1}{S} \oint_C \oint_C (\bar{a}_n \cdot \bar{a}_s)(\bar{a}_n' \cdot \bar{a}_{s'}) \left[\frac{|\bar{r}_t - \bar{r}_t'|}{d}\right]^{2n} d\ell d\ell' \tag{16}$$

$$N_{2n}^{ss'} = \frac{1}{S} \oint_C \oint_C (\bar{a}_n \cdot \bar{a}_s)(\bar{a}_n' \cdot \bar{a}_{s'}) \left[\frac{|\bar{r}_t - \bar{r}_t'|}{d}\right]^{2n} \ell n \left[\frac{|\bar{r}_t - \bar{r}_t'|}{d}\right] d\ell d\ell' \tag{17}$$

The coefficients $T_{2n}^{ss'}$ and $N_{2N}^{ss'}$ depend only on the cross-sectional shape, and not the diameter or k_c or ν. They can thus be computed once for all for a given shape (by numerical integration if by no other means) and stored for use in computing $T_{ss'}(\nu)$ for various values of $\gamma k_c d$ to find the roots of (13). In fact, analytical expressions for coefficients have been obtained for elementary shapes such as the circle, the ellipse, the semi-ellipse, or even an arbitrary polygon.

Numerical Results

We have verified that shapes such as the circle, the ellipse, and the semi-ellipse can be approximated by a polygon of 128 sides, and the results will agree to at least three digits. Hence the routines we have written for the arbitrary polygon can be used for almost any core configuration. In addition, comparisons have been made with results of other work, where available, and are shown in Figs. 2-6 . For all cases considered, $\Delta \ll 1$, and variations of Δ within this constraint did not change the plots in any perceptible way. The graphs display the value of $P^2 \equiv \gamma^2/\Delta$ vs. a normalized frequency $V \equiv k_c d\sqrt{\Delta}/2$. Results from eqn. (13) are referred to as CFA (constant field approximation).

Figure 2 shows the exact results for a circular rod, as well as the CFA when N = 5 or N = 10 terms of the series expansions (14) are retained. Agreement is seen to be quite good (within 5% or so) up to the point where the next higher-order mode appears. Figure 2 also indicates that retention of more terms than N = 10 is probably unnecessary since the CFA breaks down before this truncation does. Figure 3 is a comparison with the results of James and Gallett[15] for an equilateral triangle. Since the latter authors employ point-matching, and in view of the possible errors introduced by poor choices of matching points,[8] the greement is quite acceptable. Similar agreement with Goell's results[7] for the square and rectangle is seen in Figs. 4 and 5. These figures also display Marcatili's approximation[12] for modes away from cutoff, which is seen to complement the present results quite effectively. Finally, Fig. 6 compares the CFA with Yeh's results[5] for a weakly-guiding elliptical fiber. Even on the extremely compressed scale used by Yeh, it appears that some discrepancy exists in the single-mode range. Since Yeh's two fundamental modes are not even approximately degenerate, some doubt would seem to be present that Yeh's results indeed apply to a weakly-guiding fiber.

References

1. N.S. Kapany and J.J.Burke, Optical Waveguides. New York: Academic Press, 1972.
2. D. Marcuse, Theory of Optical Dielectric Waveguides. New York: Academic Press, 1974.
3. L.A. Lyubimov, G.I. Veselov and N.A. Bei, Radio Eng. Electron. Phys. vol. 6, pp.1668-1677, 1961.
4. C. Yeh, J. Appl. Phys. vol. 33, pp.3235-3243, 1962.
5. C. Yeh, Opt. Quantum Electron. vol. 8, pp. 43-47,1976.
6. W. Schlosser and H.G. Unger in Advances in Microwaves (L.Young, ed.), vol.1,pp.319-387. New York, Academic Press, 1966.
7. J.E. Goell, Bell Syst. Tech. J. vol.48, pp.2133-2160, 1969.
8. A.L. Cullen, O. Özkan, Electron Lett. vol.7, pp.497-499, 1971.
9. G.I. Veselov and G.G. Voronina, Radiophys. Quantum Electron. vol. 14, pp. 1482-1489, 1971.
10. C.G. Williams and G.K. Campbell, IEEE Trans.Micr. Theory Tech. vol. 22, pp. 329-330, 1974.
11. R. Pregla, AEÜ, vol. 28, pp. 349-357, 1974.
12. E.A.J. Marcatili, Bell Syst.Tech.J. vol.48, pp. 2071-2102, 1969.
13. R.M. Knox and P.P. Toulios in Proc. Symp. Submillimeter Waves, pp.497-516, Brooklyn: Polytechnic Press 1970.

14. A.P. Gorobets, L.N. Deryugin and V.E. Sotin, Radio Eng. Electron. Phys. vol.20, no.1, pp.49-55, 1975.
15. J.R. James and I.N.L. Gallett, Proc. IEE (London) vol. 120, pp. 1362-1370, 1973.
16. J.R. James and I.N.L. Gallett, Radio Electron.Eng. vol. 42, pp. 103-113, 1972.
17. C. Yeh, S.B. Dong and W.Oliver, J.Appl Phys. vol.46, pp.2125-2129, 1975.
18. T.S. Bird, Monitor, vol. 37, pp. 235-241, 1976.
19. A.D. McAulay, Int. J. Num. Meth. Eng. vol.11, pp.11-25, 1977.
20. B.Z. Katsenelenbaum, Dokl. Akad, Nauk SSSR vol. 58, pp.1317-1320, 1947.
21. C.B. Shaw, B.T. French, and C. Warner, Sci.Rept.No.2 (AD-652501) Autometics Div. of NAA,Anaheim,CA,1967.
22. C.B.Shaw and B.T. French, Final Report (AD-668553) Autonetics Div. of N.A. Rockwell, 1968.
23. D.L.A. Tjaden, Philips J.Res. vol.33, pp. 103-112, 1978.

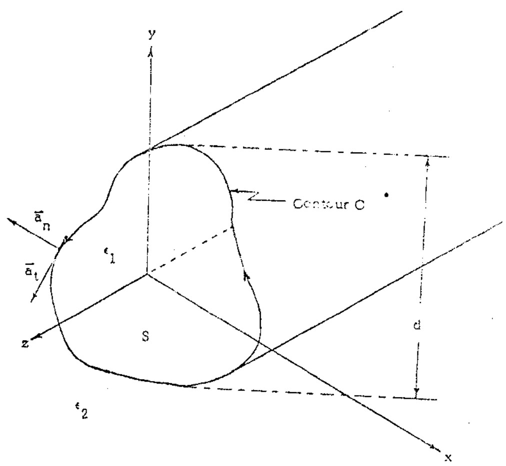
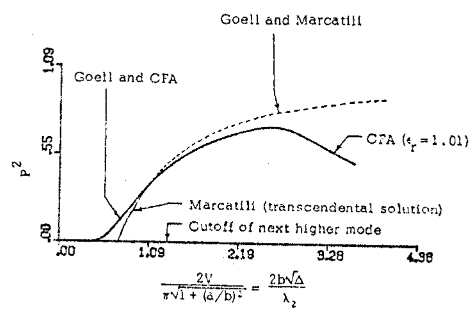
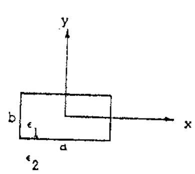
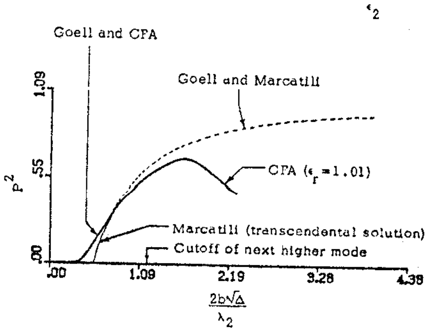


Fig. 1 Dielectric Waveguide of Arbitrary Cross Section



λ_2 = cladding wavelength





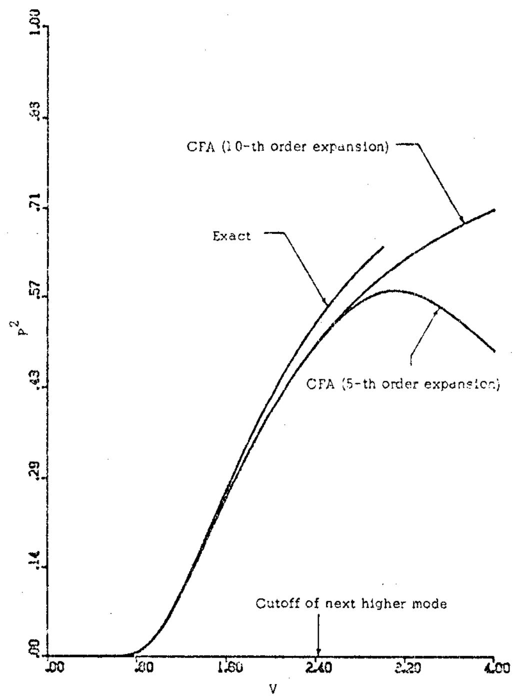


Fig. 2 Comparison with Exact Solution of Circular Cross Section (ε_r =1.01)

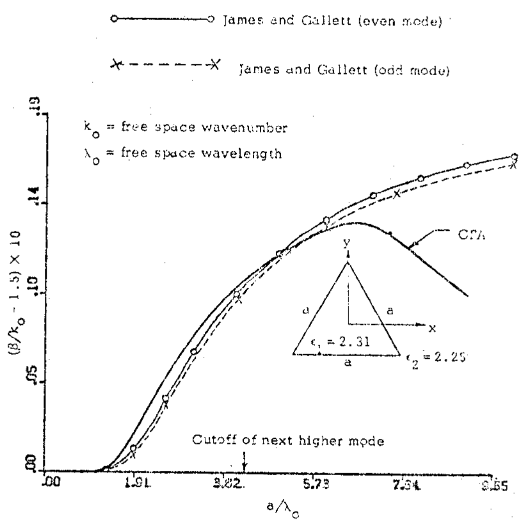


Fig. 3 Comparison with James and Gallett; Equilateral Triangle ($\varepsilon_r \cong 1.027$)

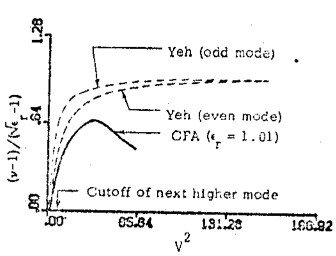


Fig. 6 Comparison with Yeh; Ellipse ($a/b \cong 2.164$, $\varepsilon_r \to 1$)

Fig.4 Comparison with Goell & Marcatili (square - $a/b = 1$; $\varepsilon_r \to 1$)

Fig.5 Comparison with Goell & Marcatili (rectangle - $a/b = 2$; $\varepsilon_r \to 1$)